

A3: Strings and QCD

Collaborative Research Centre 676

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26 Oct 2006

Staff Members and aim of the project

Strings

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QCD

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AIM

Improve our understanding of **String theory** and of **QCD** at high energy by means of the tools provided by dualities connecting gauge and string theories (AdS/CFT).

Some recent results

- Dilaton operator (anomalous dimensions) of trace operators ($\text{Tr}(ZYYZY \dots)$) in $N = 4$ SYM is equivalent to an integrable spin chain Hamiltonian;
- Similar integrable structure found in the worldsheet theory of strings on $\text{AdS}_5 \times S^5$;
- Precise identification of dual states possible: excitations on the spin chain (Magnons) are dual to certain soliton solutions in string theory.

Short distance effect (anomalous dimension \leftrightarrow leading twist) associated with the Bjorken limit: $Q^2 \propto s$, $Q^2 \rightarrow \infty$.

The Regge limit ($s \rightarrow \infty$, t fixed) offer a much richer structure. It also presents integrable structures.

The dynamics of QCD in the Regge limit simplifies largely.

ROUGH IDEA:

The light-cone ($x^\pm = x^0 \pm x^1$) and transverse ($x^i, i = 2, 3$) dynamics decouple.

- The **light-cone** dynamics provides powers of $\log(s)$ which sum up to the **Regge behaviour** s^α ;
- the **transverse** dynamics determines the **Regge trajectory** $\alpha(t)$ and “deforms” the power-like behaviour.

Perturbative approach

- If $s \gg -t \gg \Lambda_{QCD} \Rightarrow$ perturbation theory;
- The Leading Logs $(\alpha_s \log s)^n$ give the BFKL Pomeron (\mathbb{P}):

$$\mathcal{A}_{ab \rightarrow a'b'}(s, t) \xrightarrow{\text{LL}} \langle bb' | s^{1+\mathcal{H}} | aa' \rangle$$

- The **BFKL Hamiltonian** \mathcal{H} acts in the transverse space;
- It encodes two interacting **Reggeized gluons** ($\mathbb{R}\mathbb{G}$);
- $\mathbb{R}\mathbb{G}$ s are the effective d.o.f. relevant at high energy;
- They live in $2 + 1$ dimensions, $\log(s)$ plays the role of time;
- \mathcal{H} is Moebius invariant: $[\mathcal{H}, \mathfrak{sl}(2, \mathbb{C})] = 0$
(subalgebra of the Virasoro algebra);
- \mathcal{H} is holomorphic separable: $\mathcal{H} = h + h^*$.


- Next-to-LL: $\alpha_s(\alpha_s \log s)^n$.
 \mathcal{H} is known. Moebius invariance is broken due to the running of α_s . Interesting to study $N = 4$ SYM ($\beta = 0$).
- Generalized-LL: $2 \text{ RG} \rightarrow n \text{ RG}$ (BKP equations):
 - Moebius invariance: $[\mathcal{H}_n, \mathfrak{sl}(2, \mathbb{C})] = 0$;
 - **Integrable** in the large N_c limit.
- Extended-GLL: Field theory of interacting RGs in $2 + 1$ dimensions.
The known transition vertices are Moebius symmetric.

There seem to be more symmetries than those inferred from the QCD Lagrangian.

The EGLL resembles a CFT

- $\Phi_{00}(z, \bar{z}) \rightarrow \mathbb{R}G$ operator;
- $\mathcal{O}_{h\bar{h}}(z, \bar{z}) \rightarrow \mathbb{P}$ (2 $\mathbb{R}G$) operator (quasiprimary).

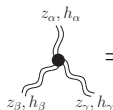
The \mathbb{P} wave function is interpreted as the correlation function:



A diagram showing a central black dot with three wavy lines extending from it. Two lines go upwards and outwards, labeled z_1 and z_2 . One line goes downwards and inwards, labeled z_0, h .

$$= \left(\frac{z_{12}}{z_{10}z_{20}} \right)^h \left(\frac{\bar{z}_{12}}{\bar{z}_{10}\bar{z}_{20}} \right)^{\bar{h}} = \langle \Phi_{00}(z_1, \bar{z}_1) \Phi_{00}(z_2, \bar{z}_2) \mathcal{O}_{h\bar{h}}(z_0, \bar{z}_0) \rangle$$

The triple \mathbb{P} vertex looks like a CFT correlation function:



A diagram showing a central black dot with three wavy lines extending from it. One line goes upwards and outwards, labeled z_α, h_α . Two lines go downwards and outwards, labeled z_β, h_β and z_γ, h_γ .

$$= V(\alpha \rightarrow \beta, \gamma) = V(h_\alpha; h_\beta, h_\gamma) \times \prod_{i < j} z_{ij}^{-\Delta_{ij}} \bar{z}_{ij}^{-\bar{\Delta}_{ij}}$$

Tempting to identify:

$$V(\alpha \rightarrow \beta, \gamma) = \langle \mathcal{O}_{h_\alpha \bar{h}_\alpha}(z_\alpha, \bar{z}_\alpha) \mathcal{O}_{h_\beta \bar{h}_\beta}(z_\beta, \bar{z}_\beta) \mathcal{O}_{h_\gamma \bar{h}_\gamma}(z_\gamma, \bar{z}_\gamma) \rangle$$

Questions

- Can we identify a CFT which describe this high energy behaviour of QCD / $N = 4$ SYM?
- Can we, using AdS/CFT, identify a dual string theory?
- Can we use this correspondence in order to improve our understanding of both QCD at high energy and of string theory?

- Let's start with $N = 4$ SYM.
 - Similar to QCD but simpler (no running of α_s);
 - $N = 4$ SYM \equiv IIB on $AdS_5 \times S^5$ is the best known example of AdS/CFT correspondence.
- Identification of suitable gauge invariant operators and computation of correlation functions in the Regge limit (different from QCD, there is no matter).
- BFKL operator at NLL known only in the forward direction ($t = 0$); Moebius invariance and integrability at NLL are still hot open issues!