Geometry in gravity and cosmology

Mattias N. R. Wohlfarth

Presentation of the Emmy Noether group

Hamburg, 30 November 2006

Members of the group

- Postdoc position offered, March 2007

- PhD student
  - Manuel Hohmann, February 2007

- Diploma students
  - Jörg Kulbartz, October 2006
  - Martin von den Driesch, November 2006

General outline

Classical gravity

$S = \int \sqrt{-g}\, R$ with $\hbar = 0$

String theory
Loop quantum gravity

?? Quantum gravity ??

$$\hbar \neq 0$$

Classical gravity

$$S = \int \sqrt{-g}\, R \text{ with } \hbar = 0$$

Classical spacetime

$\Rightarrow$ Differential geometry for the effective description of quantum phenomena

New geometry

String theory
Loop quantum gravity

?? Quantum gravity ??

$$\hbar \neq 0$$

Classical gravity

$$S = \int \sqrt{-g}\, R \text{ with } \hbar = 0$$

⇒ Generalized theories — Curvature bounds

⇒ Generalized geometry — Area geometry

⇒ String cosmology

String theory
Loop quantum gravity

?? Quantum gravity ??

$$\hbar \neq 0$$

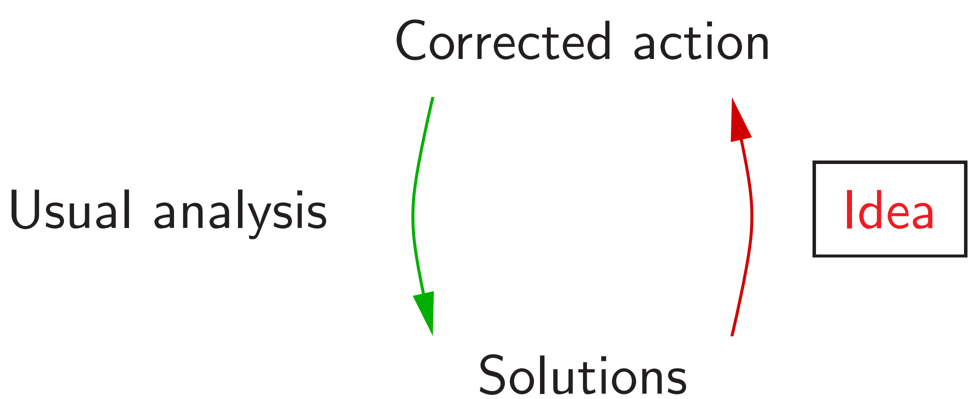
Gravity with curvature bounds

- String theory $\longrightarrow$ Einstein gravity at low energies

$$S = \int \sqrt{-g}\,(R + \lambda^2 R^2 + \ldots)\,, \quad \lambda^2 \sim \hbar$$

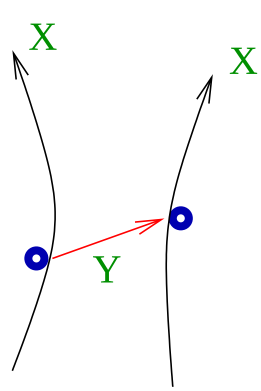
- Curvature corrections:

  How are singularities, big bang, . . . affected?



- Requirement of freedom of singularities

  $\sim$ maximal tidal (gravitational) acceleration



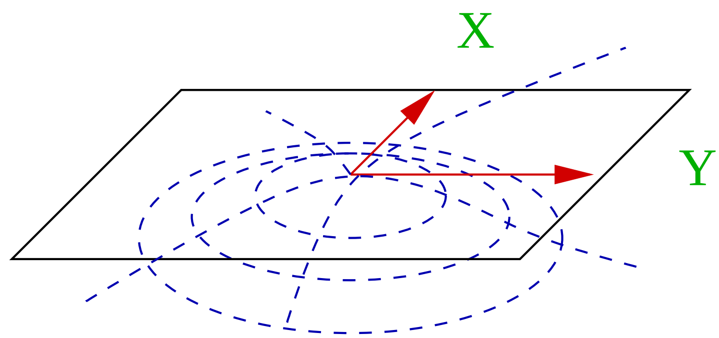
$$\text{"}\ \nabla_X \nabla_X Y = R(X,Y)X\ \text{"}$$

$$< \quad \lambda^{-2}$$

- Maximal tidal acceleration

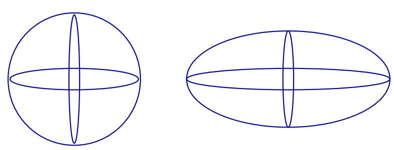
  $\sim$ maximal sectional curvature

  $$|S(X,Y)| < \lambda^{-2}$$

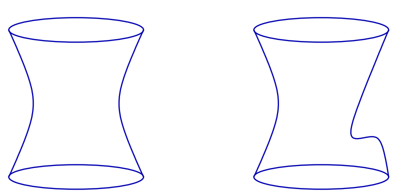


  defined for each plane $\langle X, Y \rangle$

- Riemannian geometry $\Rightarrow$ bounded sectional curvature along all planes possible

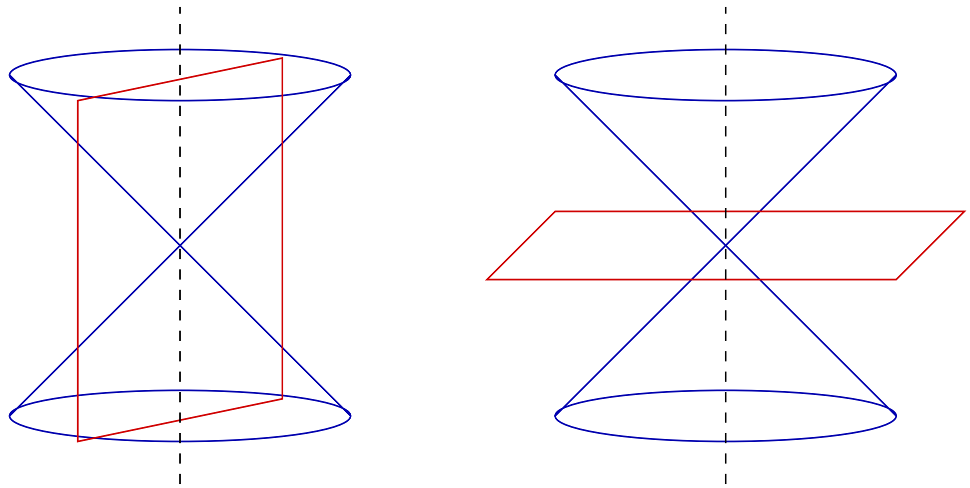


- Lorentzian geometry $\Rightarrow$ Manifolds with bounded sectional curvature have constant curvature



  Question: Gravity with curvature bounds?

- Answer: Selection of a maximal set of planes



Sub-variety of the Grassmannian

- Gravity theory (for f of finite convergence radius)

$$S = \int \sqrt{-g}\, f(R)$$

$\Rightarrow$ All solutions have maximal tidal acceleration

$\Rightarrow$ No Schwarzschild singularity

$\Rightarrow$ Completeness theorems for certain cosmologies

[Class. Quantum Grav. 21, Wohlfarth]

[Nucl. Phys. B 698, Schuller, Wohlfarth]

[Phys. Lett. B 612, Schuller, Wohlfarth]

[Phys. Rev. D 72, Easson, Schuller, Trodden, Wohlfarth]

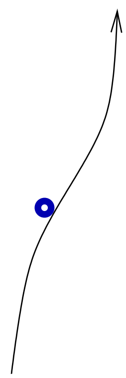
[Ann. Phys. in press, Punzi, Schuller, Wohlfarth]

- $\Rightarrow$ Petrov/ Segre classification of spacetimes with bounded sectional curvature
- $\Rightarrow$ Extension of singularity freedom in contrast to Hawking/ Penrose?
- $\Rightarrow$ Stability of the theories
- $\Rightarrow$ Analytical and numerical solutions and interpretation of initial conditions
- $\Rightarrow$ Reduction to two-dimensional dilaton gravity
- $\Rightarrow$ Applications to cosmology

Area geometry

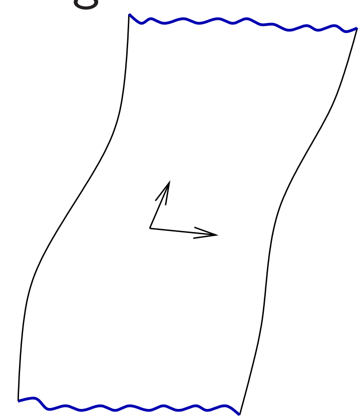
- Basic string idea:

Point particles $\Longrightarrow$ Strings



Spacetime

M

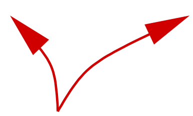


Worldlines

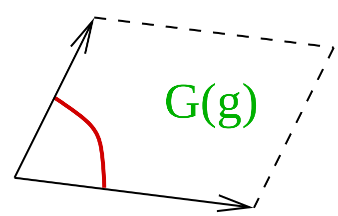
Worldsheets

- Stationarity of the action $\Rightarrow$ dynamics

Requires a measure of length/ area



Metric geometry (M, g)



- Idea

Generalized geometry

Spacetime manifold M with area measure G

Geometry $\Rightarrow$ Strings

- Variety of oriented areas $X \wedge Y$

$$A^2TM = \{\Omega \in \bigwedge^2 TM \,|\, \Omega \wedge \Omega = 0\}$$

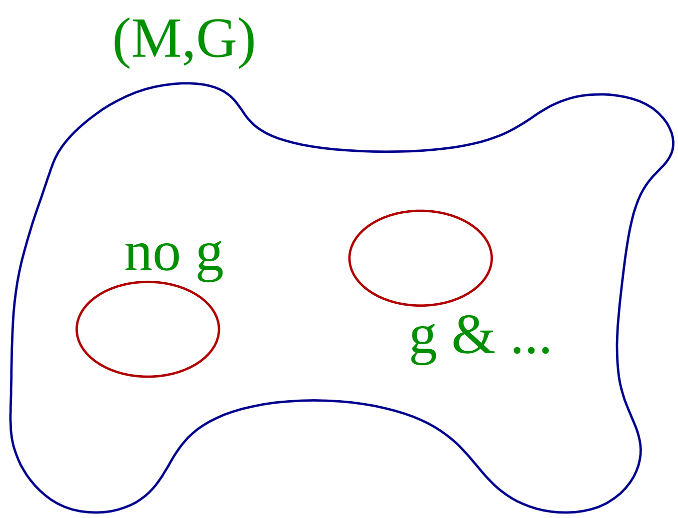
Metric on $\bigwedge^2 TM \Rightarrow$ area measure G

- G accommodates string backgrounds and D-brane geometries with g and B . . .
String motion from the minimal surface condition

[Nucl. Phys. B 747, Schuller, Wohlfarth]

[JHEP 0206, Schuller, Wohlfarth]

- Postulate: spacetime is an area metric manifold (M, G).



  - Effective metric g_G (unique in $d = 4$)
  - g_G relevant in symmetric situations (cosmology)
  - Causally well-behaved theory of gravity on (M, G) from curvature invariants
  - Comparison to Einstein gravity

[To appear soon, Punzi, Schuller, Wohlfarth]

- Area metric cosmology $+$ string fluid

  g, ϕ $\tilde{\rho}$, $\tilde{p}$, $\tilde{q}$

  $\Updownarrow$ Interpretation $\Updownarrow$

  Einstein cosmology $+$ perfect fluid

  g ϕ and $\tilde{\rho}$, $\tilde{p}$, $\tilde{q}$

  $\Rightarrow$ effective ρ, p

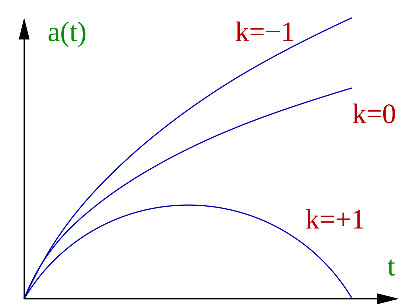
- Freedom in $w = p/\rho \Rightarrow$ realize any value (in principle)

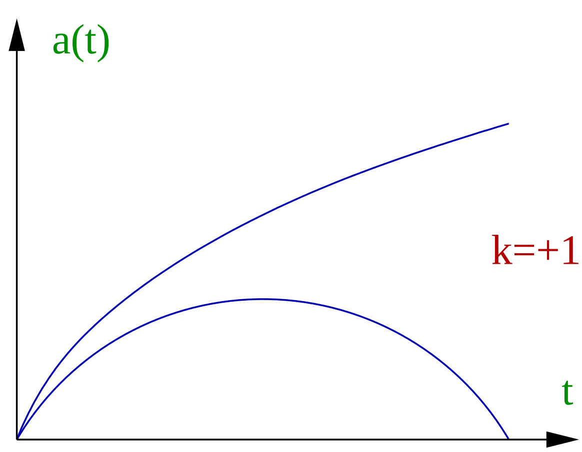
  Vacuum cosmology g, $\phi \Rightarrow$ effective radiation fluid

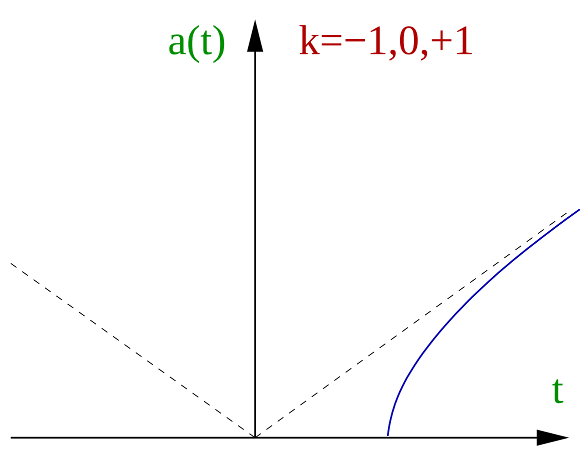
- Comparison: Einstein plus dust

  $\times$

  Late universe filled with non-interacting string dust







- Area metric cosmology $\quad + \quad$ string fluid

  $g, \phi \qquad \tilde{\rho}, \tilde{p}, \tilde{q}$

$\Updownarrow$ Interpretation $\Updownarrow$

Einstein cosmology $\quad + \quad$ perfect fluid

$g \qquad \phi$ and $\tilde{\rho}, \tilde{p}, \tilde{q}$

$\Rightarrow$ effective ρ, p

- Freedom in $w = p/\rho \Rightarrow$ realize any value (in principle)

  Vacuum cosmology $g, \phi \Rightarrow$ effective radiation fluid

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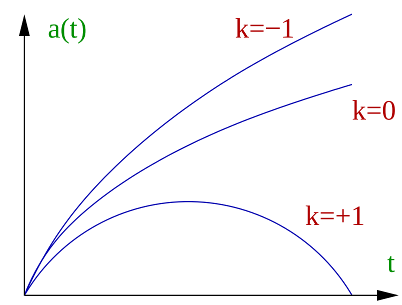
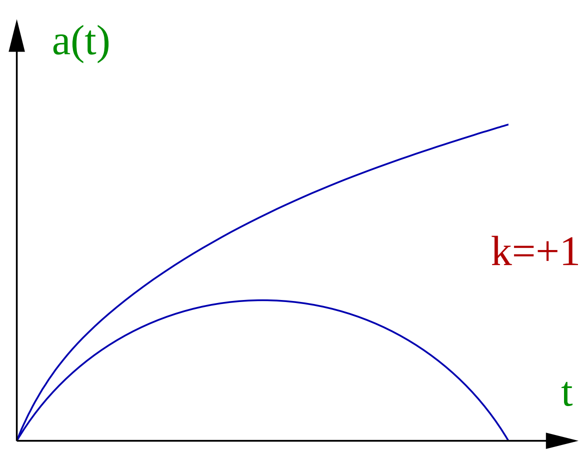
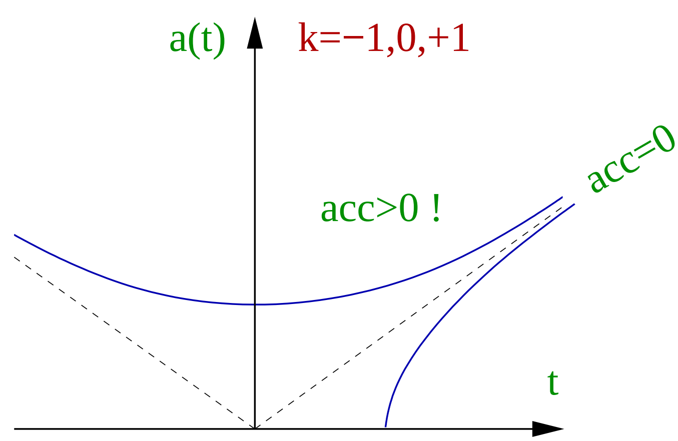






Explanation of small late-time acceleration

No dark energy, no fine-tuning

⇒ Analysis of, and relations between,
the diverse theories in dimensions $d > 4$

⇒ Solutions in spherical symmetry and singularities

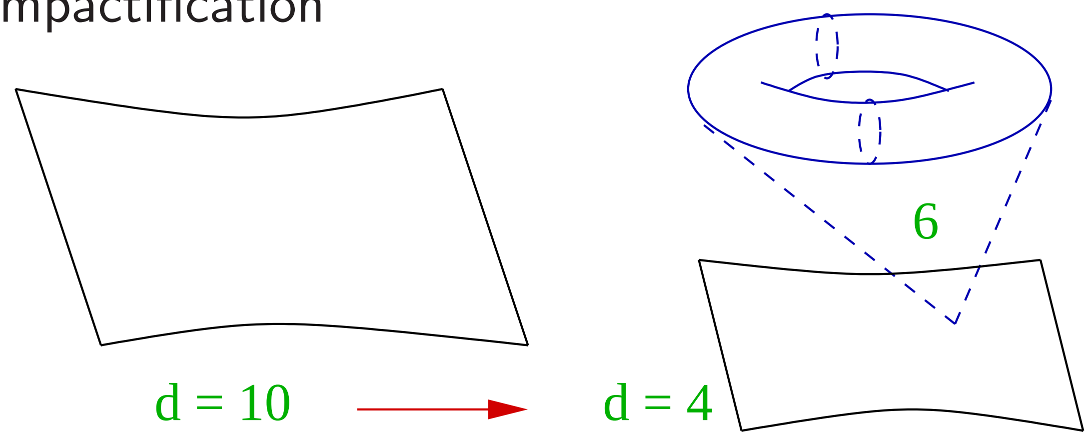
⇒ Effects seen from the Einstein perspective,
cosmology of dark energy

⇒ Development of the theory:
fermions, quantum strings

⇒ Relations to coupled spin two fields,
Regge triangulations in area variables,
generalized geometries (complex/T-folds)

String cosmology

- Compactification



⇒ Scalar fields $\sim$ moduli of the internal space

- Multi scalar cosmology

⇒ Geometric description of solutions by geodesics

[Class. Quantum Grav. 21, Townsend, Wohlfarth]

- Models with accelerating expansion

⇒ Possible by circumvention of a no-go theorem

Time dependent S-brane solutions

[Phys. Rev. Lett. 91, Townsend, Wohlfarth]

[Phys. Lett. B 563, Wohlfarth]

⇒ Universe today but no inflation

[Phys. Rev. D 69, Wohlfarth]

$\Rightarrow$ Probability measures in the space of cosmological solutions

$\Rightarrow$ S-brane solutions in conformal field theory and effective tachyon dynamics: inflation?

$\Rightarrow$ Cosmology from the classification of supergravity solutions

$\Rightarrow$ Supersymmetry breaking by cosmological fluxes: cosmological constant

$\Rightarrow$ Compactification on manifolds with non-compact isometry group

Summary

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